Comment on "Computation of Turbulent Axisymmetric and Nonaxisymmetric Jet Flows Using the K–ε Model"

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I BELIEVE that Thies and Tam[1] are probably correct in their suggestion that the difficulty in predicting the mixing in turbulent axisymmetric jets, by using the standard k–ε model, is caused by differences between the large structures in axisymmetric jets and in the flows that were used to calibrate the model initially. Contrary to the authors' claims, however, I do not think that the Pope[2] and Sarkar[3] modifications represent the correct physics of turbulent mixing in axisymmetric and three-dimensional jets. I would also suggest that the parabolic calculation procedure that they propose for three-dimensional jets may be more limited than they appear to claim and that for many applications a fully elliptic procedure may be more appropriate.

Planar/Axisymmetric Problem

Rodi[4] was probably the first to describe the difficulty of correctly predicting the spreading rate of planar and axisymmetric jets by using a two-equation turbulence model. Rodi's discussion was in the context of the k-kl model, but the k–ε model suffers from similar problems. Over the years there have been many attempts to resolve this difficulty. One of the most recent of these appeared in the December 1995 issue of this journal.[5]

The Pope modification, which Thies and Tam advocate, was first proposed in 1978 and was based on the idea that vortex stretching, due to flow divergence, could lead to increased dissipation.[2] Pope, therefore, proposed that a vortex stretching term should be added to account for this effect. On the basis of what was known at the time, this did not seem to be an unreasonable suggestion, although Bradshaw[6] had earlier pointed out that lateral divergence tended to increase the shear stress. Bradshaw's early findings, however, are now well established. Over the years, numerous comparisons of flows with and without lateral divergence have shown that lateral divergence increases rather than decreases the shear stress and turbulence level. A comparison of the planar and radial jets,[7,8] for example, shows that lateral divergence doubles the shear stress in this case, and similar effects have been found for wall boundary layer flows. So, obviously, lateral divergence cannot be the correct explanation for the slower spreading rate of axisymmetric jets.

In addition, it was found that the standard k–ε model predicted the effects of flow divergence quite well without any additional terms and that the Pope[2] modification actually led to worse rather than better predictions for such flows. Because most three-dimensional flows contain some regions with significant levels of lateral divergence, the Pope modification should generally be avoided for three-dimensional calculations. It is particularly bad for jet noise applications because a very common jet noise reduction strategy is to attempt to increase the jet mixing rate by deliberately introducing regions of lateral flow divergence to stretch the interface between the high-speed jet and the lower speed external flow. In such applications, the Pope modification will significantly degrade the ability of the k–ε model to accurately model the critical near-field region of the jet where much of the noise is produced.

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Compressibility Effects

It is true that Sarkar and others did attempt to model compressibility effects in mixing layers and jets, as the present authors propose, as a dilatational-based modification to the turbulence energy dissipation rate. But that was a long time ago. More recently,[9,10] based on detailed direct numerical simulations, Sarkar[9] has "found that the 'stabilizing' influence of compressibility in the high-speed regime is due to the reduced efficiency of turbulent production and not due to mean density variation or explicit dilatational effects." A more recent study of compressible mixing layers found "that the dilatational contribution to dissipation is negligible even when eddy shocklets are present."[11] So, whatever merits the authors' proposal may have as a purely empirical modification of the standard model, it certainly does not correctly model the flow physics of compressible jets.

Three-Dimensional Jets

Although the procedure that the authors propose for starting their jet calculations may not be unreasonable for axisymmetric jets with thin initial boundary layers, I believe that it will not work very well for many three-dimensional jets. The authors acknowledge that accurate initial conditions are desirable, but they claim that the calculations are not too sensitive to initial conditions.

I believe that it is well established that three-dimensional jets are notoriously sensitive to initial conditions and, particularly for supersonic flows, can be strongly elliptic at the nozzle exit. Reference 12 is an early example of work that describes the influence of nozzle geometry, upstream of the nozzle exit, on the subsequent development of rectangular jets. The more recent work, reported in Ref. 13, is another example of how important this effect can be. The latter work describes the development of three supersonic jets that exit from three nozzles with the same exit geometry but different upstream geometries. It is shown that the differences in the nozzle geometries upstream of the nozzle exit planes lead to significant differences in the downstream development of the three jets.

If the crossflow is sufficiently small at the nozzle exit, a parabolic analysis can give reasonable results for three-dimensional jets, but in general it is necessary to either include the crossflow at the nozzle exit,[14] and this is generally not known, or start the calculation well upstream of the nozzle exit by using a fully elliptic analysis procedure. In the past, cost considerations often necessitated the use of parabolic codes for three-dimensional jet calculations. However, this is no longer as true as it once was, and for many three-dimensional jet calculations an elliptic procedure is more appropriate, at least in the region of the nozzle exit.

There are also some issues associated with the downstream development of three-dimensional jets. For jets that are initially rectangular or elliptic, the major and minor axes generally interchange one or more times as the jet develops, before eventually becoming axisymmetric. The results of calculations of a rectangular turbulent jet, presented in Ref. 15, indicate that the tendency of rectangular jets to become axisymmetric far downstream is caused by turbulence-driven secondary flows. How important these effects will be, of course, again depends on the initial conditions but can, in some cases, be quite important. A scalar eddy viscosity of the type proposed in the referenced paper will not predict these effects.

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Reply by the Authors to S. F. Birch

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IN his comment on our paper,[1] Birch states that he does not think that the Pope[2] and Sarkar[3] modifications represent the correct physics of turbulent mixing in jets. He indicates that the modifications should not be used. However, he offers no alternative, even though it is well documented that the standard K–ε model does not predict three-dimensional/high-speed jet flows well. Further, he notes that for jets issuing from nozzles with complex geometries or complicated upstream flow conditions, accurate initial conditions for parabolized computations are important.

When Pope[2] formulated his correction term, he did appeal to the notion of vortex stretching. However, his correction may be viewed merely as a way to account for the reduction in turbulence levels in three-dimensional jet flows compared to two-dimensional shear layers. This view is the one expressed in our paper. We attribute the reduced turbulence levels to the behavior of the large-scale structures and not to some general result of flow divergence. The effect of the correction is that the spreading rates of axisymmetric and three-dimensional jets are smaller than that of the two-dimensional mixing layer. This matter was discussed in our paper. The jet flow results we obtained by using the correction are excellent when compared with experiments. We have made no recommendation regarding the use of Pope's correction in other environments (e.g., wall-bounded flows and radial jets).

In our paper, we discussed the physical mechanism[4] that causes the reduction in high-speed mixing. This mechanism is not included in the standard K–ε model. It is obvious that a modification to the K–ε model is needed if this effect is to be reproduced in the calculations. In any model, including every turbulence model, the objective is to have a simple mathematical representation that does what the physical mechanism does. We did note the fact that the Sarkar model was derived on a different physical basis. However, upon close examination, we find that the model does simulate the characteristics of the actual physical mechanism in jets. We elaborated on this issue in the paper.

A parabolized computation can be accurately applied to very complicated flows. However, it should be applied only to flows with very little upstream influence. For most jet flows, this condition is well satisfied some distance downstream of the nozzle exit. In our calculations, initial conditions are chosen to match the data at some downstream location. In each case, the data vary slowly (i.e., exhibit very little upstream influence) downstream of the first measurement point. As a result, the parabolized calculations are quite appropriate for the jet flows considered. We certainly agree that it would be foolish to use parabolized calculations for flows with significant upstream influence or for which appropriate initial conditions cannot be estimated.

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